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THE SPIKE OF COMET AREND-ROLAND 1956 h

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Abstract

The spike of April 25, 1957, is readily explained as an unusual "synchrone", consisting of dust particles of different radii released in a burst which took place between March 29 and April 2, 1957; the dust left the nucleus with almost zero relative velocity, and must have got rid of any volatile material almost instantaneously.

Radiation pressure and gravity being the only acting agents, the dust spread out into a thin, narrow and long "blade" which, when seen edgewise from the plane of the comet's orbit, appeared as the straight spike. The width of the blade in the orbital plane indicates that the duration of the main burst was about 2-3 hours only. The distance from the nucleus is indicative of the radiation pressure and radius. The radii of the dust particles ranged from 2 to 80 $\times$ 10^{-5} cm, and the frequency distribution of the radii was similar to that of zodiacal dust and very different from that of visual meteors. Explanations are suggested for the remarkable, purely radial action of radiation pressure on the individual particles.

1. *Introduction and Collected Data.* On April 25, 1957, the earth passed through the orbital plane of Comet Arend-Roland. A photograph taken by Dr. Lindsay (*Ref.* 1) with the Armagh Schmidt (scale 1 mm = 142″.6) on that date, only 4-5 hours after the passage, contains detail which may help in interpreting this unique object. The former reproduction of the photograph (*Ref.* 1) was technically somewhat unsatisfactory. In Plate II an enlarged portion of it is reproduced, showing the detail of the spike.

To the writer's knowledge, Dr. Lindsay's is the only large-scale photograph obtained on the date of passage through the plane, and its value is therefore unique [nearest to it comes the photograph with the Uppsala 30-cm Väisälä-Schmidt camera on April 25 (Ref. 16)]; in particular, its resolving power is sufficient to enable estimates to be made of the intrinsic thickness of the spike. The surface brightness of the spike decreases rapidly in a transverse direction, but effective dimensions can be assigned to it with an uncertainty of perhaps not more than ±20%. The relevant data are given in Table 1.

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The spike of Comet Arend-Roland, April 25, 1957, $22^h.5$ U.T. Armagh Schmidt exposure 20 minutes by E. M. Lindsay, enlarged 4.4×. Scale of reproduction 1 mm = 32″.4. Position of comet: $\alpha = 2^h\ 46^m.5$, $\delta = +46°19'$ (1950.0). North is above, West to the left. The total apparent length of the spike covered by the photograph is 1°.54.

Table 1

Dimensions of Spike of Comet Arend-Roland
and
Hypothetical Motion of its Particles
I. On April 25.9, 1957 (Plate II and *Ref.* 9)

1. Point	L	J	G	K
2. Source	Plate II	Plate II	Plate II	*Ref.* 9
3. Angular Distance from Nucleus	0°.14	0°.82	1°.39	15°
4. Estimated Relative Brightness of Spike	10	2.5	1.1	...
5. Angular Half-Width of Spike	25″	14″	15″	...
6. Angular Half-Width of Aureole of Spike	580″	225″	240″	...
7. Estimated Distance from Nucleus, astron. un.	0.017	0.10	0.17	0.40
8. Estimated Distance from Earth, a.u.	0.60	0.52	0.45	0.24
9. Transversal Half-Width of Spike, km	11000	5300	4900	...
10. Transversal Half-Width of Aureole, km	260000	85000	79000	...
For 23.5 days as the age of synchrone $LJGK_1$:				
11. Average Velocity of Recession from Nucleus, km/sec	1.3	7.5	13	37
12. Relative Acceleration $(1 - \beta)$	0.980	0.855	0.726	0.250
13. Radius of Stony Grains, cm	8×10^{-4}	1.1×10^{-4}	6×10^{-5}	2.1×10^{-5}
14. Average Transversal Velocity, Spike, m/sec	5.5	2.6	2.5	...
15. Average Transversal Velocity, Aureole, m/sec	130	42	40	...
II. On April 26.9 (*Ref.* 10)				
16. Width of Dense Spike in Orbital Plane, km	480000	...	200000	...
On April 27.9 (*Ref.* 11)				
17. Width of Dense Spike in Orbital Plane, km	120000	250000	330000	...
18. Total Width of Spike Fan in Orbital Plane, km	1300000	1100000	880000	...

The table is partly linked to Fig. 1. Namely, Fig 1 represents quantitative results of the calculation of the trajectories of hypothetical particles detached from the comet and moving under a decreased gravitational force. From a comparison of calculations and observations it appears that the spike corresponds more or less to a narrow " synchrone " or tail $CLJGK_1$ Ka (Fig. 1), the particles of which became detached from the nucleus between March 29 and April 2, during a short burst of 2-3 hours duration. Around April 25 at Point G the width of the synchrone was about 200,000 km in the orbital plane (16th line of Table 1), while its thickness was only about 10,000 km (9th line of the table). It thus resembled a long thin blade (CLJGK$_1$), with relative dimensions of 1 (thickness), 25 (width of denser portion) or 100 (total width of spike fan) and 6000 (length CK$_1$ = 0.4 astron. units). The great linear extension of this blade was concealed by perspective, the line of sight making a small angle with the synchrone which stretched very nearly in the direction from comet nucleus towards the earth.

From photographs obtained on later dates (*Ref.* 10, 11, and 12) it appears that the spike " was sharply bounded on the side towards the sun and more diffuse in the opposite direction " (*Ref.* 12); the sharp boundary was thus trailing behind, representing an earlier synchrone, say, aKK$_1$GC in Fig. 1, or that corresponding to anomaly of origin of —40°, whereas the diffuse portion filled the space between aKGC and the line of sight, EC, and corresponded to anomaly, say of —39°, or to a later origin. A sudden burst (fragmentation) with subsequent gradual decay (over 2-3 hours) would account well for the phenomenon.

In Table 1, the first line gives the designation of the point, in notations of Fig. 1: points L, J, and G are those from the Armagh photograph, and point K$_1$ corresponds to the greatest extension of the spike according to Fogelquist (*Ref.* 9). The other data are self-explanatory. Lines 3 to 15 are based on the Armagh photograph (Plate II) as interpreted in Fig. 1 and refer to the moment near passage through the orbital plane, i.e., to April 25.9, 1957. Guiding errors are likely to have produced a spurious half-width of the spike of about 4″ as can be judged from the stellar images in Plate II; the observed half-widths, as given in the 5th line of the table, are thus well above the resolving power of the photograph and are undoubtedly real. The relative brightness of the spike (4th line) is a rough estimate; the logarithmic scale may be in error by as much as ±20%. In the spike a dense narrow portion, or the " spike proper " appears to be surrounded by a faint " aureole ", slightly asymmetrical on the photograph (Plate II). Effective angular dimensions of spike and aureole as measured on the photograph are given in the table; these are converted into linear dimensions by assuming the geometry of Fig. 1; the freedom of geometrical interpretation is rather limited, and the absolute dimensions are hardly in error by more than 10%.

The theoretical interpretation considers a more or less sudden release of a swarm of solid particles from the nucleus of the comet; to account for the observed spike, the release has been assumed conventionally to have taken place on about April 2nd, although a somewhat earlier date (March 29) is quite probable. The particles are subject to radiation pressure, depending on size; the initial velocity of separation from the nucleus must have been small; neglecting the initial velocity, the trajectories of the particles depend solely on the effective acceleration (12th line of the table).

2. *General Interpretation.* The spike has been referred to in some publications as the " sunward tail " (*Ref.* 2, 3) or " antitail " (*Ref.* 8); this only applies to the projected appearance. From measurements of the apparent direction of the spike it is perfectly clear that, in its entirety, it was located

outside the orbit of the comet (*Ref.* 12, also *Ref.* 2, 5, 13). From *a priori* considerations it could hardly be otherwise. Forces acting on the particles, other than gravity, are believed to be those of repulsion; these would carry the particles to the outside of the cometary orbit. A considerable sunward velocity of ejection could have carried the particles slightly sunward in the beginning; however, the very small thickness of the spike layer near the orbital plane indicates that the ejection velocity was negligible or small (Table 1, lines 14 and 15), so that a true sunward tail could not have formed.

Although, in connection with the spike of Comet Arend-Roland, there have been several references to Bredichin's theory and classification of comet tails (*Ref.* 2, 3), the phenomenon was unique, not anticipated by Bredichin, and apparently never described before (*Ref.* 4). Characteristic was the concentration of the material in a thin layer, which appeared as a straight line when the earth passed through the orbital plane. Although ancient comets, e.g., those recorded in China, may have had spikes, the descriptions are so vague that no certain cases can be pointed out (*cf. Ref.* 14).

Gaseous bursts, advocated for the explanation of normal cometary tails, would lead to considerable velocity components at right angles to the orbital plane; there is no imaginable mechanism which would keep the velocities of gaseous molecules in the orbital plane. The same is true of dust particles carried away with considerable velocities by the outflowing gases. The small thickness of the layer indicates that the velocity of separation of the particles from the nucleus was very small.

This is especially true of the spike proper, where the velocities were of the order of a few metres per second only (*cf.* Table 1). We note also that, in the order of the distance from the nucleus (Points L, J and G, Table 1 and Fig. 1), while the size of the particles (Table 1, Line 13) and the effective acceleration of gravity (Line 12) are decreasing, the relative velocity of recession from the nucleus is increasing from 1.3 to 13 km/sec, essentially in proportion to the gravity defect β; the transversal velocities, however, are *decreasing* with the decreasing size of the particles, from 5.5 to 2.5 m/sec for the spike, and from 130 to 40 m/sec for the aureole. One would expect the force of repulsion, when applied to particles of asymmetrical shape, to produce ever-increasing components of velocity also at right angles to the orbital plane; the absence of the increase implies that no considerable asymmetrical action was present. The force of repulsion was almost exactly in the orbital plane, making an average angle with the latter of less than $15'$ (L), $1'.2$ (J) and $0'.7$ (G) for the spike, and of less than 6° (L), $19'$ (J) and $11'$ (G) for the aureole; these figures are estimated on the assumption that the initial velocities of separation from the nucleus were zero.

On the other hand, it may well be that the transversal velocities of the particles were almost entirely due to an initial impulse at "ejection", i.e., at separation from the nucleus; in such a case, considering that the particles had travelled in the orbital plane over an angle of more than 90°, the observed transversal velocity may amount to about two-thirds of the initial value, as would follow from the laws of Keplerian motion. This, however, will not alter our conclusions, as these depend only on the order of magnitude of the velocities.

In both cases the conclusion is that the force of repulsion which partly counterbalanced gravity was practically in a radial direction. This is a very remarkable conclusion. Whether the agent is radiation pressure, or the jet effect of the " ices " evaporating from the sunward side of the particles (Whipple's suggestion), individual grains of irregular shape will experience impulses which are radial only as an average; as a rule, lateral impulses will arise. Depending on the orientation of its axis of rotation, a single particle will be systematically deflected from a radial direction; for instance, the radiation pressure on a flat reflecting mirror rotating around its normal will always be in the direction of the axis of rotation; a population of such " mirrors " would acquire transverse velocities of the order of the velocity of recession, or several kilometres per second (Table 1, Line 11). Mineral dust of irregular or polyhedral shape would exhibit a similar property, although in a lesser degree; transversal velocities of the order of from 0.1 to 0.01 of the velocity of recession could have been expected. This, however, is not the case. The particles which formed the spike must have been highly symmetrical in shape; spherules (chondrules) of silicates, or metallic (iron) spherules, the products of condensation into a molten state from a hot atmosphere (primeval nebula) may be suggested; these spherules, later on packed into the ices condensing from a cold medium, formed part of the nucleus of the comet, whence they were released by solar heating (Whipple's theory of comet nuclei).

As to the hypothetical jet effect, we note that, even with a high reflecting power of the order of 0.8, solar radiation would have been able to evaporate a layer of 0.12 cm of water ice per hour at the actual distance of the comet's nucleus from the sun. The small detached particles (*cf.* Table 1, Line 13) would get rid of any traces of ice within a very short time, less than a minute. Whipple's jet effect (*Ref.* 2) could display itself only in the immediate neighbourhood of the nucleus, and its result, as far as the motion of the particles is concerned, will be indistinguishable from that of an initial velocity of ejection. The latter being small (as pointed out above), we conclude that the jet effect must have been negligible for the particles which formed the spike.

Thus, it appears plausible to assume that the particles of the spike became detached from the nucleus of the comet with negligible velocities, of not more than a few metres per second, and that subsequently their motion was determined by two forces only: gravitation, and radiation pressure. For the particles of the spike the latter was in a strictly radial direction, and the resultant force equivalent to an effective gravity decreased in the ratio of $1-\beta$ (Table 1, Line 12), where β is the ratio of radiation pressure to gravity. Symmetrical particles remained close to the orbital plane, producing the appearance of the spike; the less symmetrical ones may have been responsible for the aureole. Very asymmetrical particles may have been blown by radiation pressure to greater distances from the orbital plane, failing therefore to produce a concentrated layer and to impress noticeably the photographic plate.

In such a case the spike is, so-to-speak, the result of natural selection. Its sharp appearance, however, would indicate that the symmetrical particles responsible for it are remarkably numerous, representing a distinct group without a gradual transition to the less symmetrical shapes.

We note that the component of radiation pressure due to *true* absorption is always directed radially, whatever the shape of the absorbing body. On the other hand, as shown by van de Hulst, Siedentopf, and others (*cf. Ref.* 15), the reflecting power of interplanetary dust is remarkably low, possibly of the order of 0.01—0.02. If the dust in the spike possessed similar properties, the closely radial direction of the radiation pressure on individual particles is less difficult to understand; transverse impulses would be acquired only at the expense of reflected light which amounts to 1-2% of the total. With quite a considerable asymmetry in shape, partly smoothed out by rotation, the resultant of radiation pressure in individual cases still may retain a practically radial direction.

In summarizing we may say that the appearance of the spike leads to the following alternative conclusions regarding the nature of the dust particles of which it was composed:

if their albedo was not extraordinarily low, the particles must have been of symmetrical, probably spherical, shape;

if the albedo was as low as is believed to be the case with zodiacal dust, or about that of soot, the particles could have been of quite an irregular shape.

On the scanty observational evidence, it is not possible to choose between these two extreme cases, each of which contains extraordinary characteristics. However, for the analysis of the motion of the particles, the difference between these alternatives is irrelevant.

Comet Arend-Roland seems to have evolved an extraordinary amount of dust, a circumstance which undoubtedly favoured the unusual display of a spike. Thus, from microphotometer tracings of three spectrograms of the Comet, obtained in Prague on April 27, 28, and 29, Bouska and Hermann-Otavsky conclude that in the coma the integrated intensity of light reflected by dust was three times that emitted by the gaseous constituents (*Ref.* 6); also, the weak spectrum of the tail was practically a continuous one, i.e., due to reflection from dust.

3. *Dynamical Picture.* The theory of comet tails was initiated by Bessel, and later on elaborated by Bredichin. Although the spike of Comet Arend-Roland differs considerably from what Bredichin generally had in mind, some of his terminology may be usefully recalled.

The motion of the particles (molecules or dustgrains) of a comet tail is assumed to take place in the field of a central force, directed towards, or from, the sun. The most important parameter is the relative acceleration, $1-\beta$, as defined above; when β, the ratio of repulsion to gravity, exceeds unity, the net acceleration is negative; the net repulsion in the case of tails of the First Type (straight tails directed away from the sun) may be of the order of 20-40, and more, times the normal solar attraction. We note that for the spike $1-\beta$ was positive (Table 1, Line 12), or that only part of the gravity was balanced by repulsion (radiation pressure); the net force was thus one of attraction, although diminished as compared with the normal attraction of the sun.

The other parameters are the initial velocity (including direction) relative to the nucleus, and the time of separation (ejection) from the nucleus. If all these parameters were to differ within wide limits for the individual particles, their orbits would carry them at a certain moment to widely different points; the cluster of the detached particles would appear in this case as a formless diffuse nebula surrounding the nucleus, asymmetrically extending to a greater distance in a direction away from the sun and somewhat lagging behind.

Actually, comet tails are not formless structures; they exhibit characteristic shapes, different for different comets. This led Bredichin to recognize certain regularities in the formation of comet tails.

The value of the acceleration, 1-β, for a normal tail is believed to be more or less constant; gaseous molecules of a given kind would comply with this condition, as all would equally respond to the forces which act upon them in interplanetary space. The molecules separate from the nucleus in a steady flow and recede from it; those which separated earlier will have reached a greater distance than the later ones, and a tail is formed stretching from the nucleus to the outside of the orbit; the distance from the nucleus is a measure of the time of separation—the greater the distance, the earlier has separation taken place. Such a formation, produced by the action of the same force (1—β) upon a continuous flow of particles is called a *syndyname*; normal tails are syndynames which are broadened by the different (molecular) velocities and directions of ejection. Because molecular velocities are small as compared with interplanetary velocities, the resulting tail is comparatively narrow.

A sudden release of gas, or a " burst " in the nucleus will produce a cloud travelling away along the tail; because of the spread in the velocities of ejection, the particles of the cloud will arrange themselves more or less across the tail, appearing as a luminous band receding from the nucleus. Such a formation, consisting of particles released at more or less the same time, is called a *synchrone*. A syndyname consists of a succession of synchrones, all with the same value of 1—β; if the synchrones are of equal intensity, they cannot be seen separately as they merge into one more or less uniform syndyname of the tail.

The essential point in the theory of these normal tails is the assumed constancy of 1—β. On the other hand, according to Bredichin, an abnormal, truly sunward tail is formed by particles ejected sunwards, moving under a practically unaltered gravitational force, i.e., with 1—β=1.

The spike of Comet Arend-Roland does not fit into any of these models. Its interpretation as a cloud of dust particles of different size, released more or less simultaneously at an earlier date, compels one to classify it as a *synchrone;* however, from Bredichin's synchrones it differs by 1—β not being constant, and by the velocity of " ejection " being practically zero. From the present analysis it has been found that in the spike the effective acceleration, 1—β, assumed all possible positive values from 1 down to about 0.25 (Table 1, Line 12), or corresponded to variable radiation pressure on particles of different size. Actually, the distance of a particle from the nucleus depended solely on the value of 1—β, and the observed extension of the spike was mainly due to the variability of this quantity.

4. *Sample Analysis.* Assuming the orbit of the comet to be a parabola, the orbits of grains detached from the comet with zero relative velocity and subject to radiation pressure will be osculating hyperbolae (osculating = having common tangents, at the point of separation in the present case), all in the orbital plane of the comet. The hyperbolic orbits of the grains are determined in a unique manner by the point of separation and the value of $1-\beta$. Angular momentum is not influenced by radiation pressure (but for the negligible Poynting-Robertson effect); therefore, the angular momentum per unit mass of the grains is equal to that of the comet. The position of a grain at any moment of time is determined by the condition of equality of the areas swept by the radius vector of grain and comet since the moment of separation. Using this principle, calculations were made from which Fig. 1 has been constructed.

The main pattern of Fig. 1 consists of the trajectories with $1-\beta=0$, or zero effective gravity (radiation pressure balancing gravitation, a condition which would obtain for stony grains of $R = 1.6 \times 10^{-5}$ cm); the hyperbolae in this case are straight lines, tangent to the comet orbit at the point of separation. If q is the perihelion distance of the parabolic orbit of the comet, α the anomaly or the angle between radius vector and perihelion, the area of the parabola swept by the radius vector and counted from perihelion is

$$S(\alpha) = (q^2/3)\,[3\mathrm{tg}(\tfrac{1}{2}\,\alpha)+\mathrm{tg}^3\,(\tfrac{1}{2}\,\alpha)]. \qquad (1)$$

If α_1 is the anomaly at the point of separation, α_0 that of the comet on a given date, say, on April 25, the area of the parabola between α_1 and α_0 is

$$S_{10} = S(\alpha_0) - S(\alpha_1). \qquad (2)$$

The radius vector of the parabola is

$$r = q\,\sec^2\,(\tfrac{1}{2}\,\alpha). \qquad (3)$$

For the tangent trajectories with $1-\beta=0$ starting at $\alpha=\alpha_1$, $r=r_1$, the angle formed by the tangent with the radius r_1 is $90° + \frac{1}{2}\,\alpha_1$; the radius vector of the tangent at a point of separation is r_1 as defined by (3) with $\alpha = \alpha_1$, and that at anomaly α_2 is r_2, given by

$$r_2 = r_1 \cos\,(\tfrac{1}{2}\,\alpha_1)\,\sec\,(\alpha_2 - \tfrac{1}{2}\,\alpha_1). \qquad (4)$$

The area of the triangle swept by the radius vector of the tangent trajectory between α_1 and α_2 is

$$S_{12} = \tfrac{1}{2}\,r_1\,r_2\,\sin\,(\alpha_2-\alpha_1), \qquad (5)$$

or

$$S_{12} = \tfrac{1}{2}\,q^2\,\sin\,(\alpha_2 - \alpha_1).\,\sec^3\,(\tfrac{1}{2}\,\alpha_1).\,\sec\,(\alpha_2 - \tfrac{1}{2}\,\alpha_1). \qquad (6)$$

The position of the grain (r_2, α_2), simultaneous with the position α_0 of the comet on April 25, is obtained by setting equal the areas,

$$S_{10} = S_{12}. \qquad (7)$$

The solution was made in tabular form, by calculating $S(\alpha)$ from (1) and S_{12} from (6) for the following values of the arguments: α with 2° intervals; $\alpha_1 = -40°, -30°, \ldots.\ +50°, +60°$; $\alpha_2 = 50°, 54°, 58°, \ldots.\ 86°, 90°$. The solution of (7) was obtained by interpolation. From the elements of the comet (*Ref.* 7) the following data for April 25, $22^h.5$ were assumed: $q = 0.316$ astron.

un.; $r_0 = 0.595$ a.u.; $\alpha_0 = 86°.4$; $\Delta_0 = 0.622$ a.u. (geocentric distance); perihelion passage ($\alpha = 0°$), April 8.05, 1957. With these, the solutions as contained in Table 2 were obtained.

Table 2

Syndyname for Zero Acceleration ($1 - \beta = 0$) and Zero Velocity of Separation for Comet Arend-Roland on April 25.94, 1957; α_1 and α_2 are in degrees, r_2 in astronomical units

α_1	—40	—30	—20	—10	0	10	20	30	40	50	60
α_2	47.8	53.17	58.35	63.04	67.47	71.56	75.30	78.61	81.40	83.54	85.09
r_2	0.895	0.885	0.873	0.855	0.832	0.804	0.773	0.739	0.705	0.671	0.639
Point	a	b	c	d	e	f	g	h	i	k	l

The results are represented graphically in Fig. 1; the last line of the table gives the designations of the corresponding points in the figure. The figure is explained at length in its accompanying text.

The particular case of $1 - \beta = 0$ is an extreme one; as suggested in Table 1, a continuous distribution of the values of $1 - \beta$, beginning with 1 (at C, the nucleus) and decreasing with the distance from the nucleus, must have existed. The intermediate cases between $1 - \beta = 1$ and 0 correspond to curved hyperbolic trajectories. Only one such case has been calculated, that of $1 - \beta = 0.745$, $\alpha_1 = 0°$; the equation of this hyperbola is

$$r = q(1+e)/(1+e \cos \alpha), \tag{8}$$

with $e = 1.685$; the perihelion distance, q, is the same as for the parabola of the comet. In Fig. 1, this hyperbolic orbit is marked by crosses; its terminal point, H, is synchronous with C and e. Thus, CHe is a *synchrone* corresponding to ejection at $\alpha_1 = 0°$; similarly, CGa is another synchrone corresponding to $\alpha_1 = —40°$. The conditions of perspective indicate that the spike on April 25 was close to the synchrone CGa which originated at $\alpha_1 = —40°$, with R (Fig. 1) as the point of separation. This was thus the "physically occupied" synchrone, as distinct from a continuity of an infinite number of mathematically possible synchrones. The exact position of this "occupied" synchrone is irrelevant for our discussion, as all synchrones which could be observable within the sunward angle CES (Fig. 1) would lead to similar interpretations of the photographic image. We note that the measurements, by Larssen-Leander (*Ref.* 12) lead to a somewhat greater (sunward) inclination of the spike to the radius vector, as compared with that in Fig. 1; this would correspond to an earlier date of separation, about March 29.5 at $\alpha_1 = —60°$.

5. *Distribution of Particle Sizes.* The position of a particle on the synchrone is determined by the value of $1 - \beta$, the effective relative gravity. It appears to be almost certain that the force of repulsion in the present case was due to radiation pressure. If this is so, β is the ratio of radiation pressure to gravity. In the solar field and for totally absorbing, or totally reflecting spheres we may set

$$\beta = 5.63 \times 10^{-5}/(R\,\rho), \tag{9}$$

where R is the radius of the particle in cm, ρ its density in gr/cm^3. This equation is valid for $R > 1.5 \times 10^{-5}$ cm; for smaller radii there are deviations due to diffraction (*cf. Ref.* 15, p. 89); these we will not consider here, as it appears that the spike can be accounted for without postulating the presence of smaller grains.

The subsequent analysis is made to indicate a *possible* way of quantitative interpretation of the spike; there may be other possible approaches to the problem. The calculations are approximate and should rather be classified as estimates.

In Fig. 1, CHe is a synchrone with origin at $\alpha_1 = 0°$; H is the synchronous point of the hyperbolic orbit with $1 - \beta = 0.745$. Thus, on the synchrone we have three points with determined values of β and D = distance from nucleus:

Point	C	H	e
β	0	0.255	1.000
D(a.u.)	0	0.096	0.335

Along CHe, β is almost proportional to D. The empirical relation is

$$\beta \sim D^k \qquad (10)$$

with $k = 1.1$. With a good degree of approximation we can assume this to hold for other nearby synchrones, thus for CGa ($\alpha_1 = -40°$) (this saves us the calculation of intermediate hyperbolic orbits for $\alpha_1 = -40°$). With $\beta = 1$ at point a, from equation (10) with $k = 1.1$ the values of $1 - \beta$ as given in the 12th line of Table 1 have been calculated. Assuming $\rho = 3.5$ as for compact meteoritic silicates, the radii of the dust particles in the 13th line of the Table 1 are then found from equation (9). The spike turns out to be a spectrum of particle sizes, radiation pressure playing the role of a "mass spectrograph".

From the variation of brightness along the spike it seems to be possible to obtain some information on the distribution of particle sizes.

The frequency of particle radii among the material thrown into the spike can be represented by

$$\Delta n = C R^{-p} . \Delta R, \qquad (11)$$

where Δn is the number of particles with radii from R to $R + \Delta R$, p a characteristic "population exponent", and C a constant defining the total population (*cf. Ref.* 15, pp. 106, 107). For zodiacal dust $p = 2.6 - 2.85$, for the principal, so-called "E-component" of visual meteors $p = 5.2$ (*loc. cit.*).

Let ω denote the "line density", or the sum of particle cross-sections per unit length of the spike, and let j be the observed "line intensity", or apparent brightness of the spike per unit arc. We may set

$$\omega \sim j r^2 t \, \phi \sin \psi, \qquad (12)$$

where r = heliocentric distance, t = geocentric distance, ϕ = phase function of the reflected light and ψ = angle between line of sight and axis of synchrone for the given point on the spike (cosec ψ is the relative depth of the synchrone along the line of sight).

Evidently,

$$\omega \sim R^2 (\Delta n / \Delta D). \qquad (13)$$

From (10) and (9) we have

$$R \sim D^{-k}, \qquad (14)$$

whence

$$\Delta R / \Delta D \sim D^{-(k+1)}. \qquad (15)$$

Substituting (11), (15) and (14) into (13), we obtain

$$\omega \sim D^{pk-(3k+1)}. \tag{16}$$

Finally, with $k = 1.1$, from two points on the spike the value of p can be determined:

$$p = 3.91 + 0.91 \log(\omega_1/\omega_2)/\log(D_1/D_2). \tag{17}$$

The values of j were estimated on the photograph (Plate II); however crude these estimates and the corresponding values of ω, the error in p remains small, as can be seen from equation (17). Table 3 contains the relevant data. The phase law was assumed as that for the moon, and all factors are relative to those of Point L.

Table 3

Estimated Relative Optical Data for the Spike

Point (Fig. 1)	D a.u.	r a.u.	t a.u.	phase angle	ψ	j est.	ϕ lunar	$\sin\psi$ × const.	ω	R 10^{-5}cm
L	0.017	0.60	0.60	112°	6°	1	1	1	1	80
J	0.10	0.63	0.52	120°	8°	0.28	1.12	1.33	0.36	11
G	0.17	0.66	0.45	126°	11°	0.13	1.22	1.83	0.22	6

Applying formula (17) to two pairs of points, we find: from L and J, $p = 3.39 \pm 0.13$; from L and G, $p = 3.31 \pm 0.16$. The probable (or "possible") errors take into account only the estimate of $\log j$ on the photograph; the possible errors involved in the hypotheses are not included. The main uncertainty is probably in the age (α_1) and position of the synchrone, affecting the angle ψ. The total error in each value of p may be around ± 0.2. The average of the two estimates can thus be set equal to $p = 3.35 \pm 0.15$.

The population exponent, p, for the cometary dust turns out to be rather close to, and perhaps slightly greater than, that for zodiacal dust; the nearness of the values of p in the two cases appears to be significant. The exponent is definitely smaller than that for visual meteors. The increase of the number of particles with diminishing size is slightly faster for the material of the spike, as compared with zodiacal dust; in both cases it is very much slower than for visual meteors.

It may be noted that the dust released into the spike of Comet Arend-Roland is not likely to add to the zodiacal cloud; the particles, being subject to considerable radiation pressure, are moving in hyperbolic orbits and will leave the solar system for ever unless stopped by interplanetary gas. However, it appears that the only important source of interplanetary gas is thermal emission by the solar corona, or some other very soft corpuscular radiation (*cf. Ref.* 15, p. 104 and Table 4); its stopping power is too small to prevent the particles from leaving the solar system—it will, in fact, rather help in blowing the very small particles away.

In the calculations and preparation for press the author was assisted by Miss M. Walker and Mr. H. H. R. Grossie.

Armagh Observatory,
June 17, 1958.

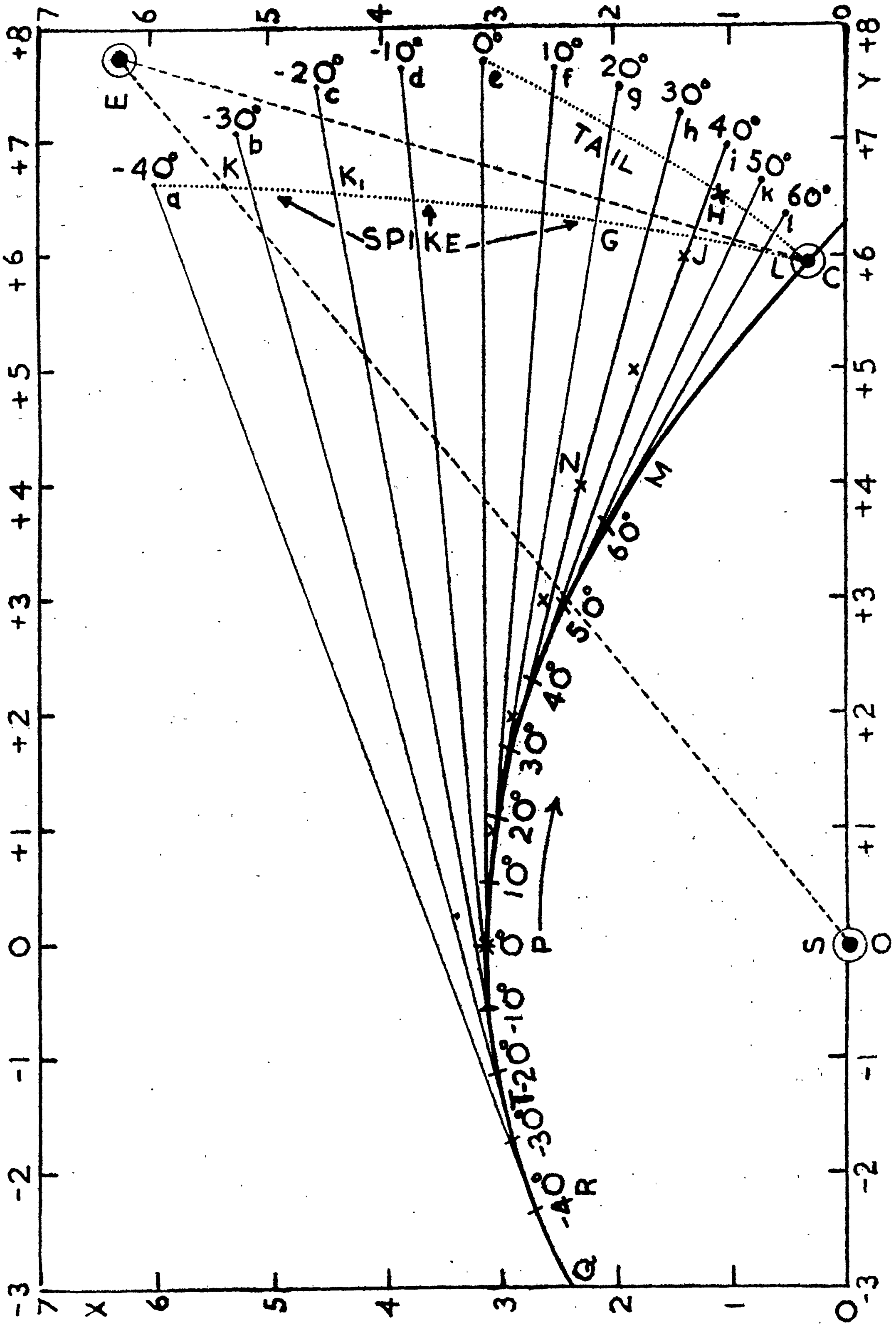
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Fig. 1

Fig. 1. Configuration in the orbital plane of Comet Arend-Roland on April 25, 1957, at $22^h.5$. Coordinates: X in the direction of the perihelion (P), Y at right angles to it; unit = 0.1 astron. units or 14,950,000 km; origin, S, in the sun. E = earth, C = comet on April 25. QRTPMC = parabolic orbit of the comet as viewed from its northern pole; the anomaly, or the angle between OP and the radius vector, is marked along the orbit (—40°, —30°......50°, 60°). The comet is viewed from earth at an apparent angular distance CES = 33°.3 from the sun. The spike is due to reflection of light from particles near the orbital plane, placed inside the angle CES (see inscription); although seen from earth sunward of the comet as the result of perspective, they actually are situated outside the orbit of the comet.

The tangent straight lines are trajectories of particles at zero gravity ($1-\beta=0$) which separated with zero relative velocity from the comet at different points of the orbit; a, b, c, d, ..., i, k, l are positions of such particles on April 25; they define a syndynamic curve (tail) for relative gravity zero. Crosses mark the hyperbolic trajectory PNH of a particle, at relative gravity $1-\beta=0.745$, which detached itself with zero relative velocity from the comet at perihelion (P, anomaly 0°); H is its position on April 25.

The dotted curve eHC is a synchrone, or the locus of particles on April 25 of relative gravity ($1-\beta$) ranging from 0 (Point e) to 1 (Point C, or the comet) which separated with zero relative velocity from the comet in perihelion (P, 0°); they are all outside the angle CES and do not contribute to the appearance of the spike. The dotted curve aKGJLC is another synchrone, or locus of particles which separated from the comet in Point R (anomaly —40°), well before perihelion passage; those along KC are placed inside the angle CES and contribute to the appearance of the sunward spike, especially over the stretch GC where they form a great thickness in the orbital plane along the line of sight from E.

From the figure it appears that only those synchrones with $1-\beta>0$ and with anomalies less than —25° (i.e. —30°, —40°, —50°, etc.) are entirely or partly inside the angle CES; therefore, the sunward spike seen on April 25 could have been formed only by particles which separated from the comet when it was at T or earlier.

Finally, attention is drawn to the Comments on p. 79 of this Journal, referring to the unusual nature of Comet Arend-Roland.

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