MATHEMATICAL MODELING OF AN ELECTROMAGNETIC FORMING SYSTEM WITH FLAT SPIRAL COILS AS ACTUATOR*

E. Paese[1], M. Geier[1], J. L. Pacheco[1], R. P. Homrich[2], J. C. S. Ortiz[1]

[1] Grupo de Projeto, Fabricação e Automação Industrial – PROMEC – Universidade Federal do Rio Grande do Sul.
[2] Laboratório de Máquinas Elétricas, Acionamentos e Energia – DELET – Universidade Federal do Rio Grande do Sul.

Abstract

This study presents mathematical modeling and calculation procedure for problems of electromagnetic forming of thin circular metal sheets using flat spiral coil as actuator. The method focuses specifically on the calculation of the electromagnetic field generated by the flat coil and analysis of the circuit that models the electromagnetic forming system. The flat coil is approximated by concentric circles carrying the current discharge from the capacitors. The calculation of electromagnetic force and magnetic couplings between the coil and metal sheet are made to the initial time, before the plastic deformation of the sheet. The method is based on the Biot-Savart law, and the solution of magnetic induction integral equations is performed by numerical methods specifically with the use of Matlab commercial software. A routine calculation, which models the problem as a set of differential equations was implemented in the Matlab, this provides important information that serves as feedback for system design. Free bulging experiments were performed to demonstrate a good relationship with the mathematical model predictions for electrical discharge current in the coil and induced currents in the metal sheet, behavior of the transient electromagnetic force between coil and workpiece and, distribution of magnetic field and electromagnetic density force along the coil. Also, achieved results showed that there is a strong dependence of the back electromagnetic force with respect to plate

---

* This work is based partially on the results of the Master of Science Dissertation of Evandro Paese. The authors would like to thank the following persons: Eng. Michael S. Ertle from TDK-EPCOS for donating the capacitors; Eng. Augusto Nienow from Ensinger Engineering Plastics for donating of polyoxymethylene block; Eng. Eliseu Silveira Brito for milling the actuator coil and dies, and to Eng. Marcio Migliavacca from Rexfort for milling the spark-gap parts.

thickness for the system analyzed. The difference phase between the current induced in the coil and workpiece with higher negative peaks generate the back electromagnetic force.

Keywords

Mathematical, Forming, Electromagnetic forming of metal sheets

1 Introduction

An electromagnetic forming system is essentially a mutual induction system composed of an actuator coil and a conductive workpiece [1]. This process is based on a repulsive force generated by the magnetic fields opposite in adjacent conductors. The transient magnetic field induces eddy currents in the metal sheet, which create a magnetic field opposite. Intense and fast repulsive forces will act on the workpiece, accelerating it at high speed [2].

Several studies have started from this premise, but most involve specific situations for deformation of tubular parts by solenoid coils, while few studies have analysed sheet metal forming by planar coils [1].

The mechanical and electromagnetic phenomena of the process are strongly interrelated, and the deformation of the sheet metal affects the magnetic field and, consequently, the Lorentz forces developed. An approximate but more realizable approach is to treat the process as a loosely coupled problem, disregarding the influence of deformation of the workpiece in the evolution of the magnetic field, and then apply the forces generated by the electromagnetic field in the mechanical problem [2].

This work will show a mathematical model of the electromagnetic forming system and numerical methods for solving a specific problem at the initial time before plastic deformation of circular metal sheets by using a flat spiral coil. The method used approximates the flat spiral coil for circular and coaxial conductors, and the sheet metal is discretized in elementary segments of circular and coaxial conductors. The magnetic field produced can be calculated by applying the law of Biot-Savart. This discretization of the workpiece allows the calculation of the electromagnetic coupling between actuator coil and workpiece. The system to be represented by a set of differential equations where the electromagnetic problem is formulated in terms of the magnetic field and the electrical problem is a circuit with mutual inductances. Experimental results are also presented for different thicknesses of aluminum plates and the results are compared with numerical solution of mathematical model in Matlab software.

2 Description of the Electromagnetic Problem

A schematic model of the system analyzed is shown in Figure 1, which shows a circular clamped metal sheet is placed above a flat spiral coil connected to a charged capacitor. The calculations of the electromagnetic problem use a method based on discretization of the metal sheet in a number of circular elementary conductors for axisymmetric configuration.

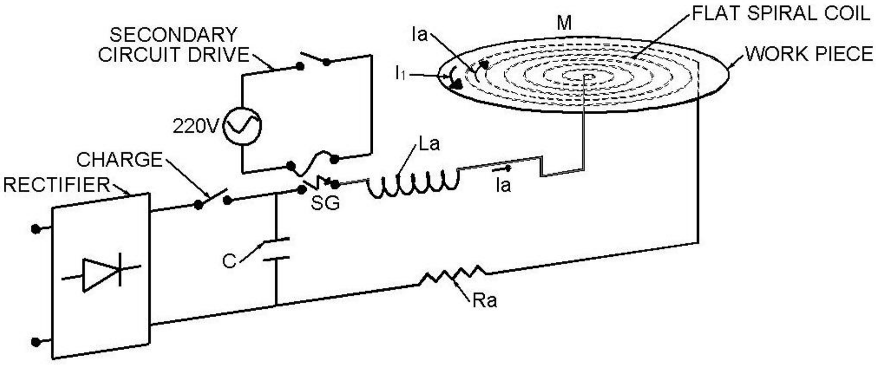


***Figure 1**: Scheme of the electromagnetic forming system.*

The transient electromagnetic problem can be separated in a RLC primary circuit coupled with secondary RL circuit [4], [5]. The discharge of the capacitor in the primary circuit can be written in differential equation:

$$\frac{d}{dt}\left(L_a \cdot I_a + M \cdot I_1\right) + R_a \cdot I_a + V_c = 0 \tag{1}$$

Where L_a, R_a and V_c are the self inductance, resistance of actuator coil and electric potential in capacitor bank. M is the mutual inductance between the actuator coil and workpiece. I_a and I_1 is the discharge current in actuator coil and the induced current in workpiece.

For secondary RL circuit the differential equation is obtained:

$$\frac{d}{dt}\left(L_1 \cdot I_1 + M \cdot I_a\right) + R_1 \cdot I_1 = 0 \tag{2}$$

Where L_1 and R_1 are the self inductance and resistance of workpiece.

This set of differential equations is solved by a resident function of the software Matlab after the discretization of the metal sheet in a finite number of conductive turns.

3 Magnetic Field

The magnetic vector potential produced by the actuator coil can be computed by applying the Biot-Savart law. The transient discretization in time is needed to evaluate the magnetic field, the density of current in the actuator coil and the workpiece and the electromagnetic forces acting on the workpiece to each variation of the time [6].

3.1 Law of Biot-Savart for Calculation of the Magnetic Field Produced by a Circular Loop at any Point in Space

Figure 2 shows a conductor with circular geometry carrying current “I”. Applying the Biot-Savart law the values of magnetic induction at the points p(x,y,z) of the space can be determined mathematically by equations (3), (4) and (5) in Cartesian coordinate system.

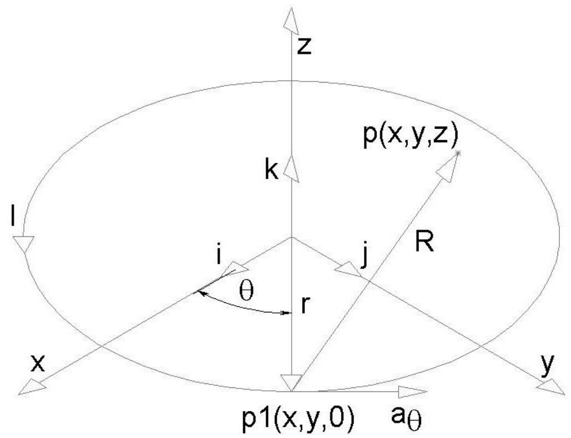


***Figure 2**: Representation of the circular conductor in cylindrical coordinates.*

$$Bx = \frac{\mu_0 \cdot I \cdot r \cdot z}{4 \cdot \pi} \cdot \int_0^{2\cdot\pi} \frac{\cos\theta}{\left[x^2 + y^2 + z^2 + r^2 - 2 \cdot r \cdot (x \cdot \cos\theta + y \cdot \sin\theta)\right]^{3/2}} \cdot d\theta \qquad (3)$$

$$By = \frac{\mu_0 \cdot I \cdot r \cdot z}{4 \cdot \pi} \cdot \int_0^{2\cdot\pi} \frac{\sin\theta}{\left[x^2 + y^2 + z^2 + r^2 - 2 \cdot r \cdot (x \cdot \cos\theta + y \cdot \sin\theta)\right]^{3/2}} \cdot d\theta \qquad (4)$$

$$Bz = \frac{\mu_0 \cdot I \cdot r}{4 \cdot \pi} \cdot \int_0^{2\cdot\pi} \frac{r - y \cdot \sin\theta - x \cdot \cos\theta}{\left[x^2 + y^2 + z^2 + r^2 - 2 \cdot r \cdot (x \cdot \cos\theta + y \cdot \sin\theta)\right]^{3/2}} \cdot d\theta \qquad (5)$$

Or in cylindrical coordinate system:

$$Br = \frac{\mu_0 \cdot I \cdot z \cdot r}{4 \cdot \pi} \cdot \int_0^{2\cdot\pi} \frac{\cos(\theta - \alpha)}{\left[x^2 + y^2 + z^2 + r^2 - 2 \cdot r \cdot (x \cdot \cos\theta + y \cdot \sin\theta)\right]^{3/2}} \cdot d\theta \qquad (6)$$

$$B_{\alpha} = \frac{\mu_0 \cdot I \cdot z \cdot r}{4 \cdot \pi} \cdot \int_0^{2\cdot\pi} \frac{\sin(\theta - \alpha)}{\left[x^2 + y^2 + z^2 + r^2 - 2 \cdot r \cdot \left(x \cdot \cos\theta + y \cdot \sin\theta\right)\right]^{3/2}} \cdot d\theta \qquad (7)$$

$$Bz = \frac{\mu_0 \cdot I \cdot r}{4 \cdot \pi} \cdot \int_0^{2\cdot\pi} \frac{r - y \cdot \sin\theta - x \cdot \cos\theta}{\left[x^2 + y^2 + z^2 + r^2 - 2 \cdot r \cdot \left(x \cdot \cos\theta + y \cdot \sin\theta\right)\right]^{3/2}} \cdot d\theta \qquad (8)$$

Where μ_0 is a magnetic constant equal to $4 \cdot \pi \cdot 10^{-7}$ [H/m].

$$\alpha = arctg\left(\frac{By}{Bx}\right) = arctg\left(\frac{y}{x}\right) \qquad (9)$$

Equations (6), (7) and (8) are used to calculate the three components of the vector density of magnetic field B at the point p in cylindrical coordinates. These equations can be solved analytically only for points located in the center of the circular geometry and p (0,0,z). This study uses resident functions in software Matlab for the numerical solution of the integral for equations above. Figure 3 shows the actual actuator coil and the simplified model with coaxial circular conductors.

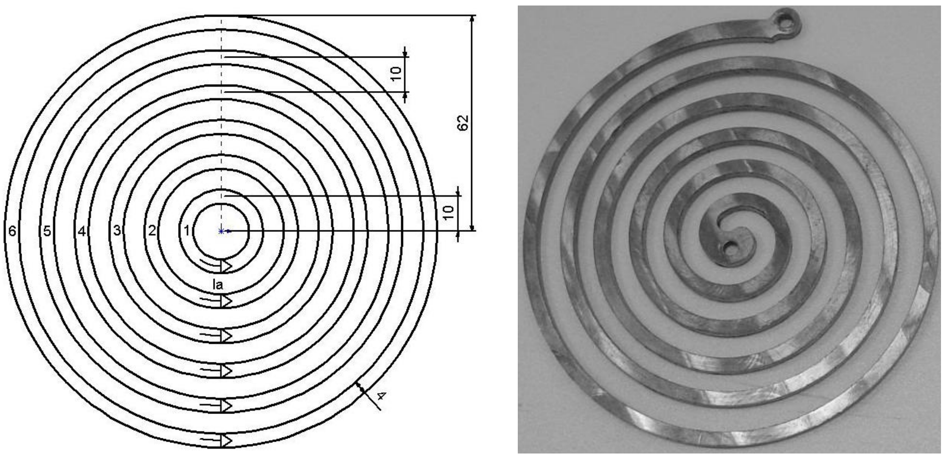


***Figure 3**: Simplified and actual model of the flat spiral coil used in this study*

Figure 4 shows the magnetic field generated by ampere in plane of the metal sheet in the radial direction. Considering the symmetry of the problem, these values are same at any radial direction.

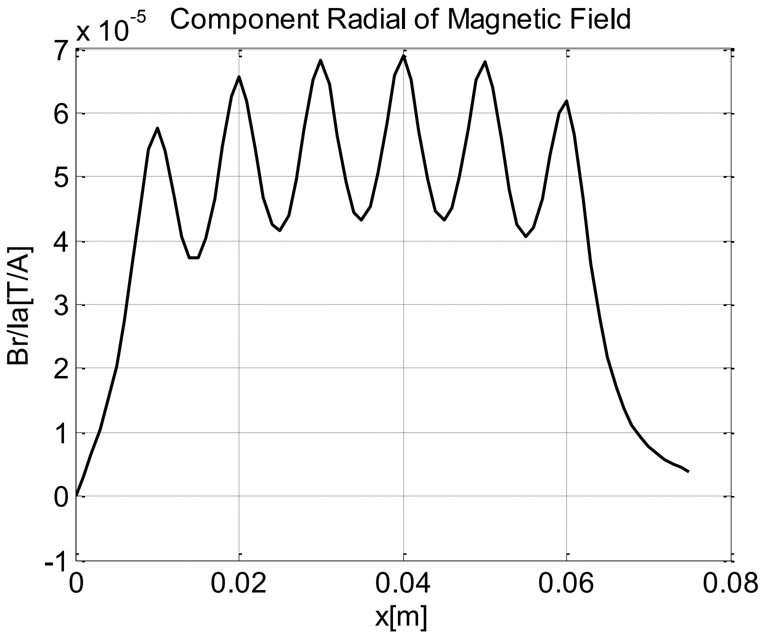


Figure 4: *Component radial of magnetic field of different thickness.*

3.2 Calculation of the electromagnetic force

The metal sheet is discretized in coaxial circular elementary conductors (L1, L2...L12), where L1 and L12 are the outer diameter and inner diameter respectively. The greater the number of circular elementary conductors the more accurate the model is but more processing is needed. The electromagnetic force generated by each circular conductor can be calculated by:

$$F_n = Br \cdot I_a \cdot I_n \cdot C_n \tag{10}$$

Where F_n, Br, I_a, I_n, C_n are the force in circular conductor n of the workpiece, the magnetic field in radial direction, the discharge current in actuator coil, the induced current in conductor n of the workpiece and length of the circular conductor n of the workpiece.

4 Parameters of the Electromagnetic Forming System

With the aid of preliminary analysis using mathematical model, a system of electromagnetic forming was developed for deep drawing of thin circular metal sheets by using a flat spiral coil. More information about the characteristics of the system are shown in Figure 5 and Table 1.

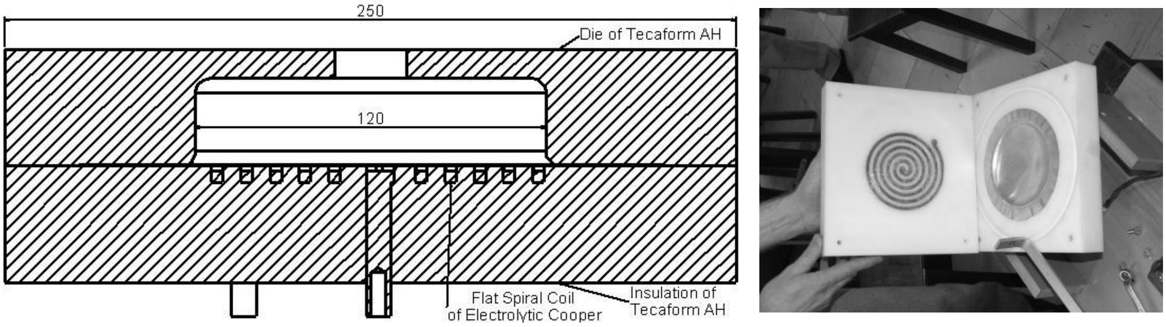


Figure 5: *Die of electromagnetic forming device.*

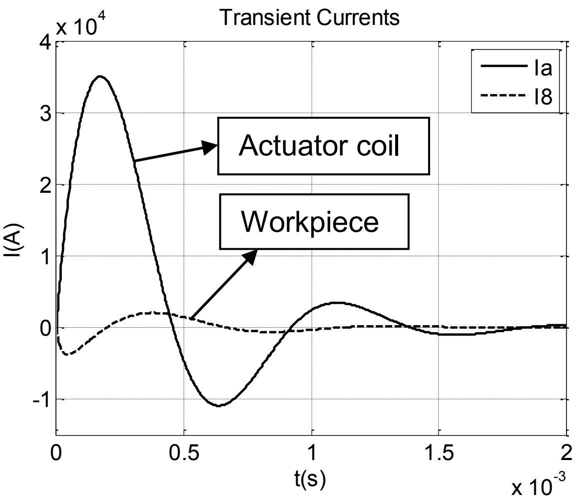
Equipment	Parameter	Value
Actuator Coil of Electrolytic Cooper	Numbers of turns	6
	Outer diameter	120mm
	Pitch	10mm
	Section of the wire	4 x 4mm
	Self inductance (L_a)	1.0326μH
	Resistance (R_a)	1.4mΩ
Bank of capacitors	Capacitance	8400μF
	Maximum voltage	900V
	Maximum energy	3.4kJ
Metal Sheet	Material	Aluminium
	Thickness analyzed	0.3, 0.5 e 1mm
	Diameter of metal sheet	170mm
	Resistivity	$2.82 \cdot 10^{-5}\,\Omega$.mm
	Gap between actuator and metal sheet	1.5mm

Table 1: System parameters and working conditions.

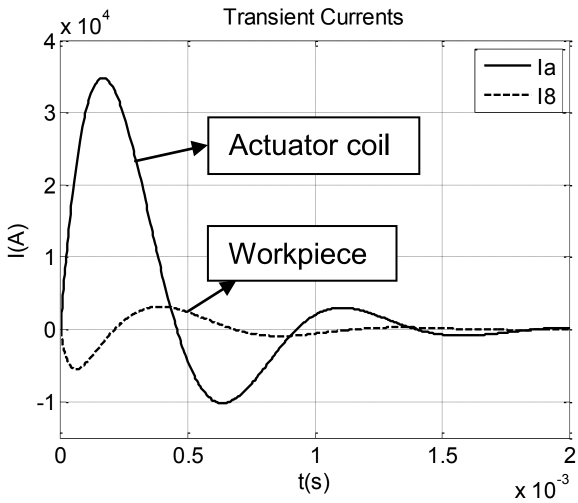
5 Numerical Simulation and Experimental Results

The system of ordinary differential equations and integrates of magnetic field was solved with numerical method in software Matlab. The component in direction z of magnetic field is important to calculate the self and mutual inductances and the component in radial direction is used to calculate the electromagnetic force in direction z. The electromagnetic system shown schematically in Figure 1 with its parameters given in Table 1, has been simulated and some of the results obtained are shown in the following figures. Figure 6 shows the simulated current in the coil (Ia) and the induced current in the workpiece that is discretized in twelve circular coaxial conductors, being that the index one is the smallest conductor. Figure 6 shows only the induced current in the conductor eight (I8), which has the highest induced current and diameter equal to 80mm.

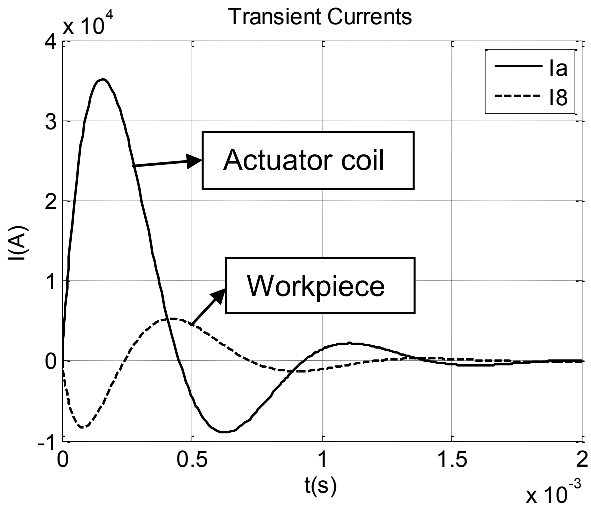
Figure 6 we can verify that the peak of electric current in the actuator remained almost constant for thicknesses of 0.3mm, 0.5mm and 1mm. The biggest changes occur in the induced currents in the elementary circular conductors of metal sheet, which shows fairly consistent, because the resistance of the elementary conductors of workpiece are strongly altered by the thickness. It is also observed that the different phase between the current in the coil and in the metal sheet is higher for thinner sheet and that the discharge occurs more rapidly when decreasing the resistance of the workpiece by increasing the thickness of it.



Thickness 0.3mm



Thickness 0.5mm



Thickness 1mm

***Figure 6**: Simulated currents in the actuator coil and in the workpiece.*

The system behaves as a RLC circuit underdamped and the currents in the source coil and workpiece are opposite. The first peak of the induced currents are around -0.38kA, -0.55kA and -0.83kA. Figure 7 is shows the axial maximum transient electromagnetic force and we can see that considerable forces are developed in this process and that more difference phase between the current in the actuator coil and currents in workpiece causes a greater back electromagnetic force when it's compared with electromagnetic force of repulsion. This shows that back electromagnetic force is more in thinner metal sheets.

The energy for total deep drawing of plates analyzed by conventional methods is equal to 237, 503, 1582 Joules [7]. Comparing these values with the energy stored in capacitors shows that much of this available energy already is dissipated in the equivalent resistance of primary circuit.

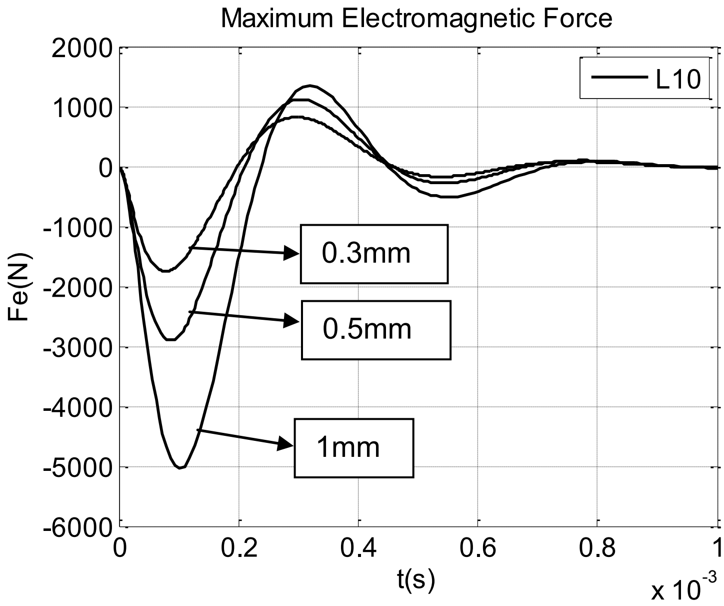


***Figure 7**: EM forces in the actuator coil and in the plate.*

Figure 8 shows the distribution of the peak repulsive force for axisymmetric problem. Figure 9 shows experimental results for different thicknesses.

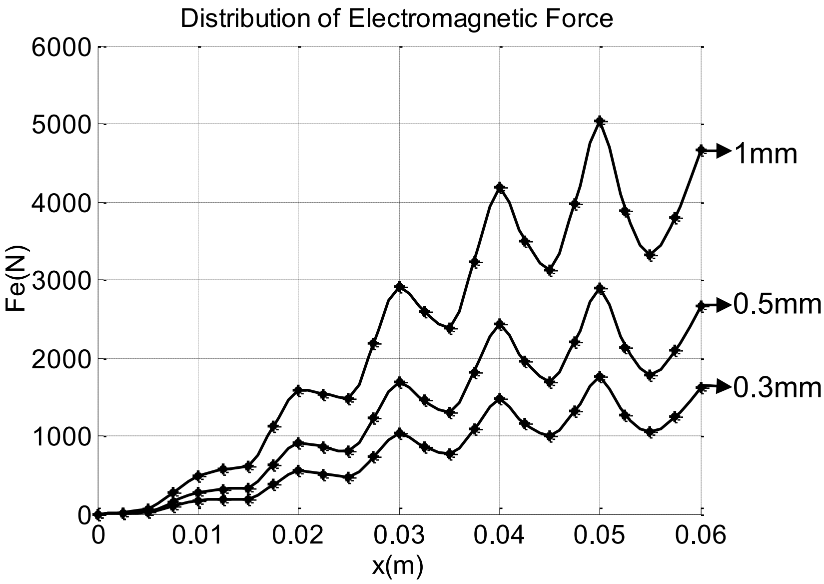


***Figure 8**: Distribution of the peak repulsive electromagnetic force.*

Thickness 0.3mm Thickness 0.5mm Thickness 1mm

***Figure 9**: Experimental results for different thicknesses.*

6 Conclusions

It can be observed in practical experiments that the back electromagnetic force in thinner metal sheets made it unable to form electromagnetically with the system proposed in this paper. This work developed a numerical method which can simulate the electromagnetic forming of metal sheets in initial time before plastic deformation. This algorithm was implemented in software Matlab. This mathematic model showed satisfactory results when compared with experiments, validating it and the method of numerical solution. In this algorithm the process parameters can be easily changed to obtain better results, and theories used in the construction of this algorithm are easily evidenced in it. This facilitates the understanding of the electromagnetic problem and results obtained are satisfactory and can be used for design flat spiral coils. This mathematical modeling was used as a basis for designing an experimental machine for electromagnetic forming.

References

[1] *Takatsu N., Kato M., Sato K., and Tobe T.:* High speed forming of metal sheets by electromagnetic force. J.S.M.E. International Journal, vol. 31, no. 1, p. 142-148, 1988.

[2] *Mamalis, A. G., Manolakos, D. E., Kladas, A. G., Koumoutsos, A. K., Ovchinnikov, S. G.*: Electromagnetic forming of aluminum alloy sheet using a grooved die: numerical modeling. The Physics of Metals and Metallography, Vol. 102, Suppl. 1, p. S90–S93, 2006.

[3] *Boutana, I., Mekideche, M. R.*: Simulation of aluminum sheet electromagnetic forming with several dies. Systems, Signals and Devices, 2008. IEEE SSD 2008. 5th International Multi-Conference on, p.1-6, July 2008.

[4] *Kamal, Manish*: A uniform pressure electromagnetic actuator for forming flat sheets. Dissertation of Graduate Program in Materials Science and Engineering School of the Ohio State University, 2005.

[5] *Shang, Jianhui*: Electromagnetically assisted sheet metal stamping. School of The Ohio State University, 2006.

[6] *Meriched, A., Feliachi, M., Mohellebi, H.*: Electromagnetic forming of thin metal Sheets. Magnetics, IEEE Transactions on, vol.36, no.4, p.1808-1811, 2000.

[7] *Lange, K.*: Handbook of Metal Forming. Michigan, Society of Manufactures Engineers (SME), 1994.

[8] *Paese, E.*: Electromagnetic Forming of Thin Metal Sheets: Technical Feasibility. MSc Thesis (in Portuguese), Universidade Federal do Rio Grande do Sul, Brazil, 2010.

[9] *Xu, W.; Fang, h.; Xu, W.*:Analysis of the Variation Regularity of the Parameters of the Discharge Circuit with the Distance Between Workpiece and Inductor for Electromagnetic Forming Processes. journal of materials processing technology 203 (2008) 216–220.

[10] http://www.epcos.com/inf/20/30/db/aec_09/B43456__B43458.pdf, last access on 02/23/2010.